Oblique Half-Solitons and their Generation in Exciton-Polariton Condensates

H. Flayac, D.D. Solnyshkov, and G. Malpuech
Clermont Université, Université Blaise Pascal, LASMEA, BP10448, 63000 Clermont-Ferrand, France and CNRS, UMR6602, LASMEA, 63177 Aubière, France

We describe oblique half-solitons, a new type of topological defects in a two dimensional spinor Bose Einstein condensate. A realistic protocol based on the optical spin Hall effect is proposed toward their generation within an exciton-polariton system.

Introduction. Exciton-polaritons (polaritons) are hybrid quasi-particles that share the properties of both light and matter. On one hand, their excitonic part allows them to interact both with themselves and with the surrounding phonon bath. In microcavities, this peculiarity leads to efficient relaxation processes towards the ground state and to the formation of a Bose-Einstein condensate (BEC)[1] with extended spatial coherence[2]. On the other hand, the photonic part of polaritons attributes them a very small effective mass which allows condensation even at room temperature[3].

While the polariton BEC has indeed been obtained experimentally, the superfluidity, namely the appearance of a dissipationless flow, was still an issue. Strong progress has been made in 2009 when the behavior of a flow of polaritons past an obstacle was analyzed[4, 5]. One of the fascinating properties of a BEC is the possible creation of topological defects by a perturbation. In spinor polariton condensates, elementary excitations are peculiar: indeed, two exciton spin projections ($\sigma^{\pm} = \pm 1$) can efficiently couple with light. Thus the wave function (order parameter) of a polariton BEC is no more scalar but vectorial. In such a framework elementary topological defects carry half-integer angular momentum and are called half-vortices (HV). These objects were predicted theoretically[6] in a thermodynamic limit, analyzed in presence of various (effective) magnetic fields[7–9] and observed experimentally[10] as a result disorder in microcavities.

A soliton is another example of topological defect that can basically be seen as a 1D analogue of a vortex. It is characterized by a maximum phase shift of π (dark soliton), on the scale of the healing length ξ of the condensate, accompanied by a ditch in the density profile. In 2D, a new type of extended defect has been recently described: oblique dark solitons (ODS)[11], which were shown to be persistent or rather only convectively unstable[12]. They can be generated if a supersonic condensate flow hits a defect larger than ξ[11, 13]. These predictions have been recently verified in a polaritonic system[14]. Polaritons can also exhibit bright solitons[15], but these are not in the focus of the present Letter. On the other hand, solitons in spinor 1D condensates[16, 17] and oblique solitons in spinor 2D systems have also been theoretically considered[18]. In one dimension, many possible configurations were described, depending on the strength and type of the particle interactions (repulsive or attractive). In particular, a solution where the kink lies in only one component was reported: the dark-antidark soliton. This kind of defect reminds the HV configuration and we will see why it deserves to be called a "half-soliton"[19].

In the present work we discuss the generation of oblique dark half-solitons (ODHS), in a spinor BEC. The paper is organized as follows. First, we analyze the 1D half-soliton by analogy with the half-vortex. Second, we introduce the ODHS. Third, we propose two experimental configurations leading to their formation, capitalizing on the specificities of polariton condensates. The first protocol is using a defect acting only on one circular component. The second one is using the TE-TM splitting present in planar microcavities.

The 1D half-soliton. A two components polariton condensate at 0K can be described by a spinor Gross-Pitaevskii equation[20]. Assuming first a parabolic dispersion with an effective mass m^* and an infinite lifetime of the particles, one has

$$i\hbar\frac{\partial\Psi_\sigma}{\partial t} = -\frac{\hbar^2}{2m^*}\Delta\Psi_\sigma + \left(\alpha_1|\Psi_\sigma|^2 + \alpha_2|\Psi_{-\sigma}|^2\right)\Psi_\sigma \quad (1)$$

Time-independent solutions are found upon expressing the BEC wave functions as $\Psi_\pm(\mathbf{r},t) = \psi_\pm(\mathbf{r})\exp(-i\mu t)$, where $\mu = (\alpha_1 + \alpha_2)n_0$ is the chemical potential related to the density of the homogeneous condensate $n_0 = n_{0+}/2 = n_{0-}/2$. Indeed, for a realistic polariton BEC, the interactions between the particles of opposite spin are weak and attractive: $\alpha_2 \simeq -0.2\alpha_1$ which leads to a linearly polarized ground state. When looking for a single-kink solution such as a vortex or a soliton, it is convenient to discuss the asymptotic behavior. We assume that far away from the defect the condensate density is unperturbed.

The order parameter can be written in two distinct representations[6, 8]. On the basis of circular polarizations discussed previously, each component possesses its own phase $\theta_\pm(\mathbf{r})$ and $(\psi_+, \psi_-) = \sqrt{n_0}/2\left(e^{i\theta_+}, e^{i\theta_-}\right)$. In the linear polarization basis, the polarization angle $\eta(\mathbf{r})$ and the global phase of the condensate $\varphi(\mathbf{r})$ can be separated: $(\psi_x, \psi_y) = \sqrt{n_0}\left(e^{i\varphi}\cos(\eta), e^{i\varphi}\sin(\eta)\right)$. The transformation from one to another is obtained via $\psi_\pm = (\psi_x \mp i\psi_x)/\sqrt{2}$ and $\theta_\pm = \varphi \mp \eta$. In the latter form, half-vortices are characterized by two half-integer

winding numbers for both the phase and the polarization which lead to integer shifts of π around their core. Similar considerations can be applied to solitons. Indeed, in one dimension the scalar dark soliton solution[21] is simply given by $\psi_S(x) = \sqrt{n_0}\tanh(x)$ and its phase is a Heaviside function of amplitude π. In the circular polarization basis and, in the simplest case where $\alpha_2 = 0$, a half-soliton (HS) is nothing but a usual soliton appearing in one component (let's say ψ_-) while the other remains homogeneous. Thus, the associated order parameter reads $\left(\psi_+^{HS}, \psi_-^{HS}\right) = \sqrt{n_0/2}\,(1, \tanh(x))$. Rewriting the latter on the linear polarization basis leads to $\left(\psi_x^{HS}, \psi_y^{HS}\right) = \sqrt{n_0}/2\,(1+\tanh(x), i - i\tanh(x))$. Looking at asymptotic forms, one easily obtain

$$\begin{aligned}\psi_x^{HS}(+\infty) &= \sqrt{n_0}e^{2ih\pi}\cos(2s\pi)\\ \psi_y^{HS}(+\infty) &= \sqrt{n_0}e^{2ih\pi}\sin(2s\pi)\\ \psi_x^{HS}(-\infty) &= \sqrt{n_0}e^{ih\pi}\cos(s\pi)\\ \psi_y^{HS}(-\infty) &= \sqrt{n_0}e^{ih\pi}\sin(s\pi)\end{aligned} \tag{2}$$

with h and s half-integer numbers which can be seen as topological charges. Basic HS appear for $(h,s) = \{(\pm1/2, \pm1/2), (\pm1/2, \mp1/2)\}$ and their phase and polarization angle are ranging from 0 to $\pi/2$ with a fully circularly polarized center just like the half-vortex core. This topological defect can also be seen as a domain wall with respect to x- and y-polarized particles. A plot of the HS density profiles ($n_j = \left|\psi_j^{HS}\right|^2$, $j = \pm, x, y$) is proposed in Fig.1(a).

The oblique half-soliton. For a 2D condensate it was shown[11, 12, 18] that stationary extended oblique dark solitons can occur both in the scalar and spinor systems despite their instability, because the latter is only convective. They appear in a perturbed fluid at supersonic velocities as merging vortices arranged in "trains" ("streets").

Let us now focus on the possibility of creating ODHS. First of all, it is clear that in the case where the two components of the spinor BEC do not interact ($\alpha_2 = 0$), if they are initially equally populated, a perturbation in only one of the components will lead to the formation of half-integer excitations. Next, what happens if the interaction is no longer negligible? To answer this question we turn back to Eqs.(1) and (2), rescale them, look for stationary solutions and switch to the hydrodynamic picture of Ref.18 where the phase of each components is expressed by means of their stationary and irrotational velocity fields via $\mathbf{v}_\pm(\mathbf{r}) = \hbar/m^*\boldsymbol{\nabla}\theta_\pm(\mathbf{r})$, with $\mathbf{r} = (x, y)$. We look for oblique solutions that depend only on the tilted coordinate $\chi = (x - ay)/\sqrt{1+a^2}$ which leads to the following set of equations

$$\begin{aligned}\left({n'_\sigma}^2/4 - n_\sigma n''_\sigma/2\right) + 2n_\sigma^2\left(\Lambda_1 n_\sigma + 2\Lambda_2 n_{-\sigma}\right)&\\ = (q+2\mu)\,n_\sigma^2 - qn_0^2&\end{aligned} \tag{3}$$

where $\Lambda_{1,2} = \alpha_{1,2}/(\alpha_1+\alpha_2)$ and $q = U^2/(1+a^2)$ (U the velocity of the flow). This system has to be solved numerically, but let us start with some simple arguments. The density profile of an integer oblique dark soliton[11] is given by $n_{ODS} = 1 - (1 - q/\mu)\,\mathrm{sech}[\chi\sqrt{\mu - q}]^2$ with $\mu = \Lambda_1 n_0$. Now, for the case of the ODHS, the density notch in the σ^- component, that contains the defect, is seen as an external potential by the initially unperturbed σ^+ component, because of the interactions between the particles of different spins. We suppose that the σ^+ component fits the shape of this potential which is nothing but $\Lambda_2 n_-$. Then, this perturbation creates in turn a potential for the σ^- component given by $-\Lambda_2 n_+ = -\Lambda_2^2 n_-$. Therefore, the density profile is modified as $\widetilde{n}_- \leftarrow \left(\Lambda_1 n_- - \Lambda_2^2 n_-\right)/\Lambda_1$. Iterating this procedure leads to a geometric series and to a renormalization of the interaction constant $\widetilde{\Lambda}_1 \leftarrow \Lambda_1 - \Lambda_2^2/(\Lambda_1 - \Lambda_2)$. Consequently, the ODHS solution is approximated by

$$n_{ODHS} = 1 - (1 - q/\widetilde{\mu})\,\mathrm{sech}\left[\chi\sqrt{\widetilde{\mu} - q}\right]^2 \tag{4}$$

with $\widetilde{\mu} = \widetilde{\Lambda}_1 n_0$. In this description, the sound velocity is changed like $c_s \to \widetilde{c}_s = \sqrt{\widetilde{\mu}/m^*}$ and the healing length like $\xi \to \widetilde{\xi} = \hbar/\sqrt{2m^*\widetilde{\mu}}$. In the case where $\Lambda_2 < 0 (> 0)$, which corresponds to an attractive (repulsive) interaction, c_s is slightly increased (decreased) and inversely for ξ. The component without a soliton obviously presents a minimum (maximum). This argumentation is compared to direct numerical solutions of Eqs. (4) and (5) in the

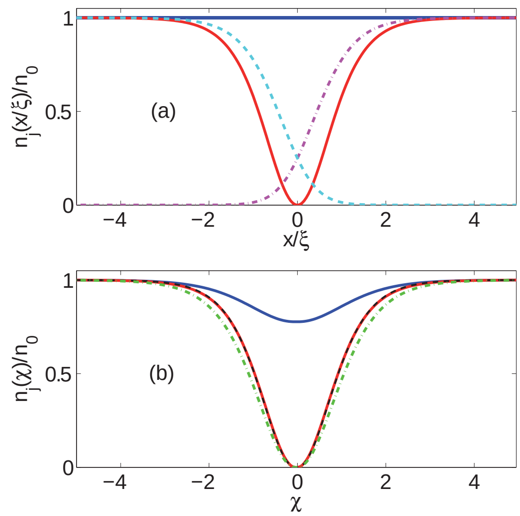

FIG. 1. (Color online) (a) HS density profiles scaled to n_0 and ξ. The solid blue and red curves represent the n_+ and n_- density profiles while the dashed-dotted purple and dashed cyan curves show n_x and n_y respectively. (b) ODHS density profiles normal to its axis. The solid blue and red curves show numerical profiles, the dashed black curve is the perturbative solution described in the text and the dashed dotted green curve show the scalar soliton solution. The parameters are $a = 10$, $U = 3.5$, $\Lambda_1 = 1$ and $\Lambda_2 = -0.2$.

Fig.1(b) showing a remarkable accuracy for small values of Λ_2.

Generation of ODHS in a polariton fluid. Bringing an external perturbation in a quantum fluid is an efficient way to nucleate topological defects in a controlled manner. A relevant experimental setup relies on the acceleration of a superfluid above the sound velocity past an obstacle larger than ξ, where the breakdown of superfluidity allows the elementary excitations to develop. The defect is a potential barrier resulting from an off-resonance laser beam. A planar microcavity is a system that offers a high degree of control on the velocity and the density of 2D spinor BECs as discussed in Refs.14 and 22.

Now, to describe more accurately the polariton condensate, we take into account the real non parabolic dispersion of the particles via a set of four coupled spin-dependent equations for the photonic $\Psi_{\pm}^{ph}$ and excitonic $\Psi_{\pm}^{ex}$ fields respectively. A resonant pumping situation is considered with photon lifetime of 100 ps.

$$
\begin{aligned}
i\hbar\frac{\partial \Psi_{\pm}^{ph}}{\partial t} &= -\frac{\hbar^2}{2m_{ph}^*}\Delta\Psi_{\pm}^{ph} + D_{\pm}\Psi_{\pm}^{ph} + \frac{\Omega_R}{2}\Psi_{\pm}^{ex} \\
&\quad - \frac{i\hbar}{2\tau_{ph}}\Psi_{\pm}^{ph} + P_{\pm} + \beta\left(\frac{\partial}{\partial x} \mp i\frac{\partial}{\partial y}\right)^2\Psi_{\mp}^{ph} \quad (5)\\
i\hbar\frac{\partial \Psi_{\pm}^{ex}}{\partial t} &= -\frac{\hbar^2}{2m_{ex}^*}\Delta\Psi_{\pm}^{ex} + \frac{\Omega_R}{2}\Psi_{\pm}^{ph} \\
&\quad - \frac{i\hbar}{2\tau_{ex}}\Psi_{\pm}^{ex} + \left(\alpha_1\left|\Psi_{\pm}^{ex}\right|^2 + \alpha_2\left|\Psi_{\mp}^{ex}\right|^2\right)\Psi_{\pm}^{ex} \quad (6)
\end{aligned}
$$

The new quantities that appear are a quasi-resonant *cw* pumping which is taken to be linearly polarized $P_+ = P_-$, the strong coupling between exciton and photons with a Rabi splitting $\Omega_R = 10$ meV and the photonic TE-TM splitting[23] of strength β for which we assume a quadratic dependence over the wave vector[8]. We assume that the potential barrier we consider $D_{\pm}$ can affect independently each circularly polarized photonic component.

In the numerical experiments that follow we reproduce the protocol described in Refs.13 and 14 involving an immobile defect and a moving fluid. The pump laser is tuned slightly above the bare polariton branch (+0.3 meV) to remain resonant despite the interaction-induced blue shift. The spot (15 μm upstream from the defect) is bar shaped ($x \times y = (10 \times 100)\mu m^2$). The fluid propagates from the left to the right with a wave vector $k_{p,x} = 1\mu\text{m}^{-1}$, low enough to avoid parametric instabilities, and collides against a 6 μm large circular defect. Defining the sound velocity is far from trivial so far as the local pumping and propagation effects induce density fluctuations and its general exponential decrease. The sound velocity $c_s = \sqrt{\mu/m^*}$ of an unperturbed flow is expected to decrease like $\sqrt{\exp\left(-m^* d/\hbar k\tau_{ph}\right)}$ where $m^* \sim 2m_{ph}^*$ and d is the distance from the pump. Strictly speaking, c_s has to be defined locally and separately for each components. We choose a pseudospin representation, which is relevant to provide information about both components of the order parameter in a single picture. We recall that the pseudospin $\mathbf{S}(\mathbf{r}, t)$ is a 3D vector lying on a Poincaré sphere. Its components are defined by

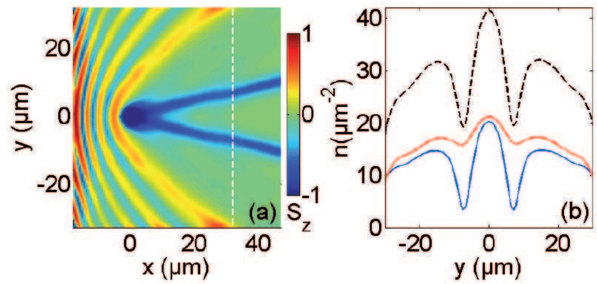

FIG. 2. (a) Pseudospin representation of the order parameter at $M \sim 1.8$. The colormap shows the S_z component (degree of circular polarization). (b) Density slice displaying n_+ (blue curve) where the soliton appears, n_- (red curve) with a minimum at the soliton position and the sum of the two (dashed black curve). The dashed white lines indicate the slice position.

$$
\begin{aligned}
S_x &= \Re\left(\psi_+^{ph}\psi_-^{ph*}\right)\\
S_y &= \Im\left(\psi_+^{ph*}\psi_-^{ph}\right) \quad (7)\\
S_z &= \left(n_+^{ph} - n_-^{ph}\right)\Big/2
\end{aligned}
$$

normalized to unity. $S_{x,y}$ define linearly polarized states while S_z is the degree of circular polarization of the condensate[23]. Below, we propose two different experimental configurations for the generation of ODHS.

The first alternative is to find a way to perturb only one component. This can be done for polaritons where the defect potential could be optically created by a circularly polarized pulse[24]. This scheme is however far from being ideal since it would bring of lot of unwanted perturbations to the system and it should rather be seen as a model experiment. In this framework, we impose $D_+ = 20$ meV, $D_- = 0$, and $\alpha_2 = -0.2\alpha_1$. The $S_z(x, y)$ component of the corresponding stationary flow regime, is presented in Fig.2(a) for a Mach number $M \sim 1.8$. The generated pair of oblique solitons is left circularly (σ_-) polarized and with the help of a density slice normal to the flow Fig.2(b) we are able to undoubtedly identify an ODHS.

Our second proposal is, on the other hand, completely realistic. The impenetrable defect is restored in both components and we suggest to benefit from the spatial separation brought by the $\mathbf{k}$-dependent TE-TM splitting. The strength of the induced effective magnetic field $\mathbf{\Omega}_{LT}$

is given by $\beta = \hbar^2 \left(m_l^{-1} - m_t^{-1} \right) /4$ where $m_{l,t}$ are the effective masses of TM and TE polarized particles respectively. The energy splitting between the two latter eigen modes is $\Delta_{LT} = \beta k_{p,x}^2 = 0.02$ meV for $m_t = 5 \times 10^{-5} m_0$ and $m_l = 0.95 m_t$ (m_0 is the free electron mass). Moreover, the field makes a double angle with respect to the direction of propagation and is thus symmetric with respect to the x-axis. The latter leads to the so-called optical spin Hall effect: the precession of the pseudospin around $\mathbf{\Omega}_{LT}$[25, 26]. In our numerical setup, the linearly x-polarized injected particles are initially moving along the x-direction so no polarization separation is expected until the fluid reaches the obstacle. Indeed, at this precise moment, the flow is split into two parts moving in oblique directions. These two parts start to precess about the effective field in opposite directions. Particles going to the left gain a σ^+ component. Particles going to the right gain a σ^- component. This antisymmetry results in the formation of an oblique soliton in one circular component, and not in the other, which is an ODHS. The figure 3(a) shows the resulting polarization pattern achieved in the steady state: A pair of oblique dark solitons of opposite circularity is formed behind the obstacle. It should also be noted that the $\tilde{\mathrm{C}}$erenkov radiations exhibit a remarkable polarization separation upstream from the defect. The figure 3(b) shows a density slice of each circular component that reveals the ODHS pair. The profiles are qualitatively similar to the one obtained for the ideal case presented on the figure 1(b). The 3(c) shows the results of a similar calculation, but for a splitting twice larger. One can clearly see the formation and disappearance of a pair of ODHS close to the defect, and, further away, the formation of a second one of opposite circularity. The figure 3(d) shows the density when the four ODHS are simultaneously present. This behavior is due to the TE-TM splitting which couples the circular components and therefore couples ODHS of opposite circularity. To finish, we make the following remark: In the transient regime, before the steady state is established or at lower fluid velocities, we have clearly observed the generation of HVs, and it will be the topic of a future work.

Conclusions. We have reported a new type of half-integer topological defect that occurs in two-components 2D BEC: the oblique dark half-soliton. We have shown that an exciton-polariton fluid is well-suited for the study of such defects. We have made a realistic proposal, based on the optical spin Hall effect, which should lead to their experimental observation. Besides, the setup we suggest should also allow a controlled nucleation of half-vortices, which was never reported so far.

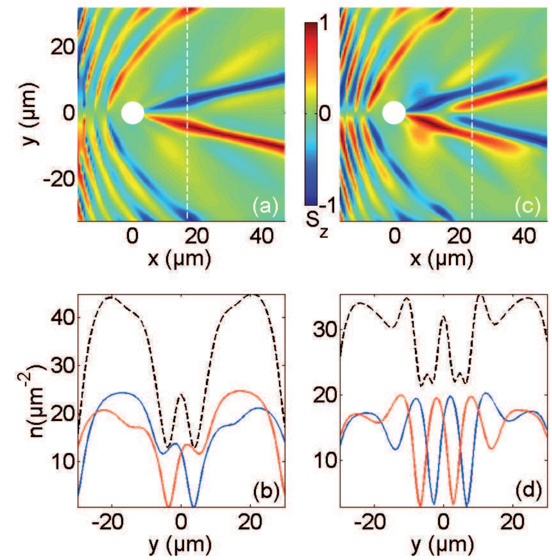

FIG. 3. Same representations as Fig.2. ODHS pair(s) in the presence of the TE-TM splitting. (a)-(b) $\beta k_{p,x}^2 = 0.02$ meV and (c)-(d) $\beta k_{p,x}^2 = 0.04$ meV.

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